A set $\Omega$ in $\mathbb{R}^d$ is called spectral if the space $L^2(\Omega)$ admits an orthogonal basis consisting of exponential functions. Which sets $\Omega$ are spectral? This question is known as "Fuglede's spectral set problem".

In the talk we will be focusing on the case of product domains, namely, when $\Omega = A \times B$. In this case, it is conjectured that $\Omega$ is spectral if and only if the factors $A$ and $B$ are both spectral. We will discuss some new results, joint with Nir Lev, supporting this conjecture, and their applications to the study of spectrality of convex polytopes.